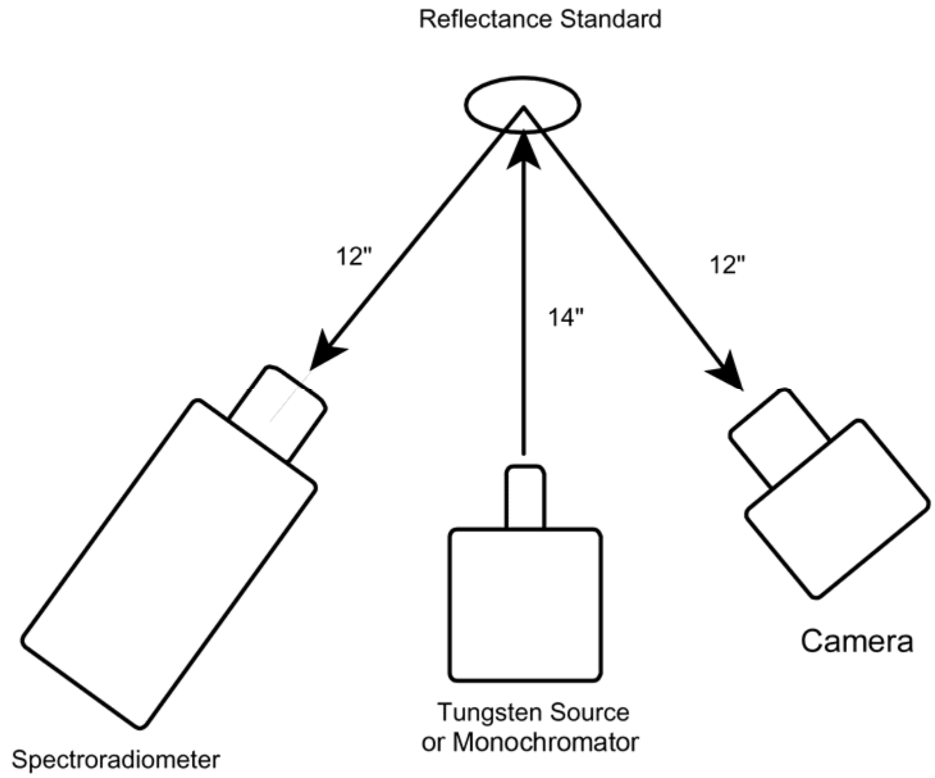
Camera Calibration Notes
(10/02/09, 6/1/14)



Suppose that the camera *F-number* is set to *f*, and that the shutter duration was set to *T* (in seconds). The effective aperture area when $f = 1.0$ is

$$a_0 = \frac{K}{1.0^2}$$

where *K* is a lens-dependent constant. The aperture area for an arbitrary *F-number* is

$$a = \frac{K}{f^2}$$

The relative area A is given by

$$A = \left(\frac{K}{f^2}\right) / \left(\frac{K}{1.0^2}\right) = \frac{1}{f^2} \tag{1}$$

Measurements for monochromatic light

For each of a large number of monochromator settings we have the following.

Monochromator with white test plate (Labsphere certified reflectance standard):

$L_e(\lambda;\lambda_0)$ (units are $w \cdot m^{-2} \cdot \omega^{-1}$)

Spectroradiometer response (the total radiance and the wavelength of the peak):

$$\hat{L}_e(\hat{\lambda}_0) = \int_{-\infty}^{\infty} L_e(\lambda;\lambda_0) d\lambda$$

Camera:

Average raw R, G, B values in a patch corresponding to spectroradiometer aperture:

$$R(\hat{\lambda}_0) = A(\hat{\lambda}_0) T(\hat{\lambda}_0) K_R \int_{-\infty}^{\infty} L_e(\lambda;\lambda_0) S_R(\lambda) d\lambda$$

$$G(\hat{\lambda}_0) = A(\hat{\lambda}_0) T(\hat{\lambda}_0) K_G \int_{-\infty}^{\infty} L_e(\lambda;\lambda_0) S_G(\lambda) d\lambda$$

$$B(\hat{\lambda}_0) = A(\hat{\lambda}_0) T(\hat{\lambda}_0) K_B \int_{-\infty}^{\infty} L_e(\lambda;\lambda_0) S_B(\lambda) d\lambda$$

$$R(\hat{\lambda}_0) = K_R \hat{S}_R(\hat{\lambda}_0) \hat{L}_e(\hat{\lambda}_0) A(\hat{\lambda}_0) T(\hat{\lambda}_0)$$

$$G(\hat{\lambda}_0) = K_G \hat{S}_G(\hat{\lambda}_0) \hat{L}_e(\hat{\lambda}_0) A(\hat{\lambda}_0) T(\hat{\lambda}_0)$$

$$B(\hat{\lambda}_0) = K_B \hat{S}_B(\hat{\lambda}_0) \hat{L}_e(\hat{\lambda}_0) A(\hat{\lambda}_0) T(\hat{\lambda}_0)$$

Thus, the estimates of the camera spectral sensitivities are:

$$\hat{S}_R(\hat{\lambda}_0) = \frac{R(\hat{\lambda}_0)}{\hat{L}_e(\hat{\lambda}_0) A(\hat{\lambda}_0) T(\hat{\lambda}_0) K_R} \quad (2)$$

$$\hat{S}_G\left(\hat{\lambda}_0\right) = \frac{G\left(\hat{\lambda}_0\right)}{\hat{L}_e\left(\hat{\lambda}_0\right)A\left(\hat{\lambda}_0\right)T\left(\hat{\lambda}_0\right)K_G} \quad (3)$$

$$\hat{S}_B\left(\hat{\lambda}_0\right) = \frac{B\left(\hat{\lambda}_0\right)}{\hat{L}_e\left(\hat{\lambda}_0\right)A\left(\hat{\lambda}_0\right)T\left(\hat{\lambda}_0\right)K_B} \quad (4)$$

where these spectral sensitivities have each been normalized to a peak of 1.0 by dividing by appropriate constants K_R, K_G, K_B .

Example of using these formulas

$$\int L_e(\lambda)S_R(\lambda)d\lambda = \frac{f^2}{TK_R}R \quad (5)$$

$$\int L_e(\lambda)S_G(\lambda)d\lambda = \frac{f^2}{TK_G}G \quad (6)$$

$$\int L_e(\lambda)S_B(\lambda)d\lambda = \frac{f^2}{TK_B}B \quad (7)$$

where *f* is the current *F-number* and *T* is the shutter duration in seconds.

To get the luminance, for example, we want:

$$Y = 683\int \bar{y}(\lambda)L_e(\lambda)d\lambda$$

We first find the weights w_{RY}, w_{GY}, w_{BY} such that to best approximation (see Fig. 1):

$$\bar{y}(\lambda) = w_{RY}S_R(\lambda) + w_{GY}S_G(\lambda) + w_{BY}S_B(\lambda) \quad (8)$$

Once we have these weights then using equations (2)-(4) we have approximately:

$$Y = 683\frac{f^2}{T}\left(w_{RY}\frac{R}{K_R} + w_{GY}\frac{G}{K_G} + w_{BY}\frac{B}{K_B}\right) \quad (9)$$

For X, Y, Z, assuming color matching functions as modified by Judd (1951) and Vos (1978):

$$\begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} = \begin{pmatrix} w_{RX} & w_{GX} & w_{BX} \\ w_{RY} & w_{GY} & w_{BY} \\ w_{RZ} & w_{GZ} & w_{BZ} \end{pmatrix} \begin{pmatrix} S_R(\lambda) \\ S_G(\lambda) \\ S_B(\lambda) \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{683 f^2}{T} \begin{pmatrix} w_{RX} & w_{GX} & w_{BX} \\ w_{RY} & w_{GY} & w_{BY} \\ w_{RZ} & w_{GZ} & w_{BZ} \end{pmatrix} \begin{pmatrix} K_R^{-1} R \\ K_G^{-1} G \\ K_B^{-1} B \end{pmatrix} \qquad (10)$$

For L, M, S, assuming Stockman & Sharpe (2000) cone fundamentals:

$$\begin{pmatrix} \bar{l}(\lambda) \\ \bar{m}(\lambda) \\ \bar{s}(\lambda) \end{pmatrix} = \begin{pmatrix} w_{RL} & w_{GL} & w_{BL} \\ w_{RM} & w_{GM} & w_{BM} \\ w_{RS} & w_{GS} & w_{BS} \end{pmatrix} \begin{pmatrix} S_R(\lambda) \\ S_G(\lambda) \\ S_B(\lambda) \end{pmatrix}$$

$$\begin{pmatrix} L \\ M \\ S \end{pmatrix} = \frac{683 f^2}{T} \begin{pmatrix} w_{RL} & w_{GL} & w_{BL} \\ w_{RM} & w_{GM} & w_{BM} \\ w_{RS} & w_{GS} & w_{BS} \end{pmatrix} \begin{pmatrix} K_R^{-1} R \\ K_G^{-1} G \\ K_B^{-1} B \end{pmatrix} \qquad (11)$$

One way to find the optimal weights is to fit each desired fundamental (e.g., obtain a least squares fit of the right side of equation (8) to the true $\bar{y}(\lambda)$ function).

From the calibration data for the Nikon D700 with 50 mm Sigma prime lens:

$$(K_R, K_G, K_B) = (1.85E8, 1.18E8, 1.47E8) \qquad (12)$$

The linear transformation matrices are given by (see Fig. 1):

$$\begin{pmatrix} w_{RX} & w_{GX} & w_{BX} \\ w_{RY} & w_{GY} & w_{BY} \\ w_{RZ} & w_{GZ} & w_{BZ} \end{pmatrix} = \begin{pmatrix} 1.137 & 0.162 & 0.141 \\ 0.586 & 0.908 & -0.170 \\ 0.034 & -0.153 & 1.464 \end{pmatrix} \qquad (13)$$

$$\begin{pmatrix} w_{RL} & w_{GL} & w_{BL} \\ w_{RM} & w_{GM} & w_{BM} \\ w_{RS} & w_{GS} & w_{BS} \end{pmatrix} = \begin{pmatrix} 0.807 & 0.821 & -0.161 \\ 0.247 & 0.984 & -0.098 \\ 0.022 & -0.108 & 0.855 \end{pmatrix} \qquad (14)$$

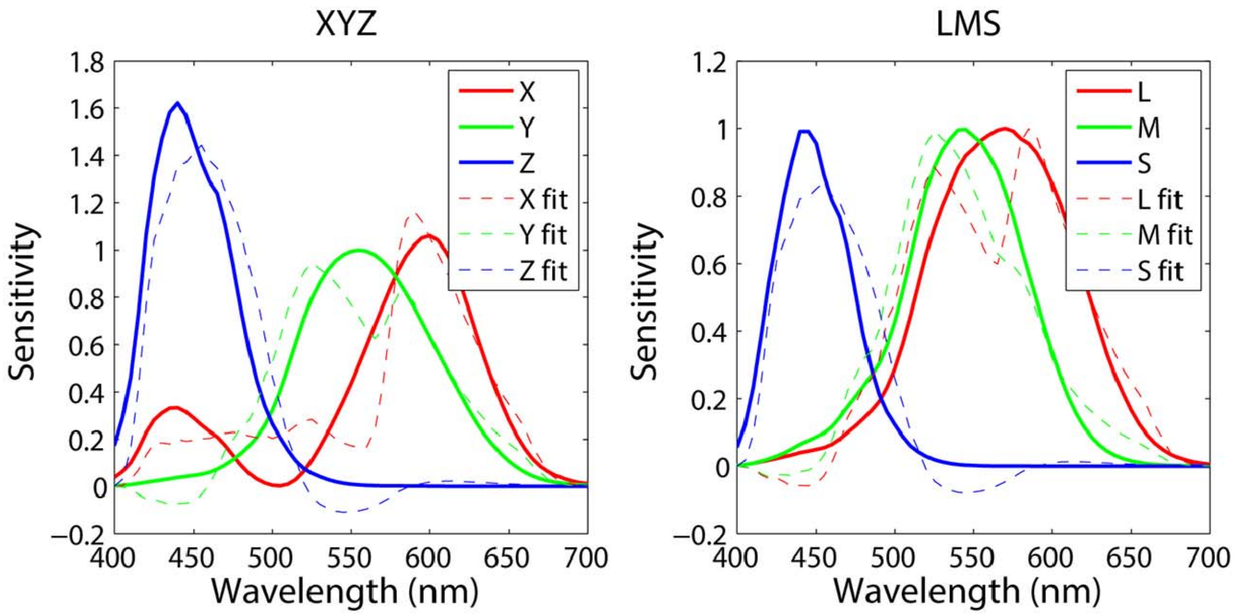


Fig. 1 Least squares fit of camera spectral sensitivities to the XYZ and LMS fundamentals.

Finally, incorporating the scalar constants into the weights we have

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{683 f^2}{T} \begin{pmatrix} 6.155E-09 & 1.376E-09 & 9.558E-10 \\ 3.174E-09 & 7.723E-09 & -1.152E-09 \\ 1.819E-10 & -1.300E-09 & 9.951E-09 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix} \quad (15)$$

$$\begin{pmatrix} L \\ M \\ S \end{pmatrix} = \frac{683 f^2}{T} \begin{pmatrix} 4.370E-09 & 6.984E-09 & -1.096E-09 \\ 1.338E-09 & 8.373E-09 & -6.669E-10 \\ 1.185E-10 & -9.217E-10 & 5.814E-09 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix} \quad (16)$$

The values of the *F-number* (f) and the exposure time in seconds (T) are available in the EXIF file for each image.

Additional Notes

Retinal quantum irradiance (derived from Wyszecki & Stiles 1982)

$n_\lambda = t(\lambda)\frac{\lambda T_\lambda}{V(\lambda)}2.649\times 10^4$ quanta per second per square millimeter per nm ($s^{-1}\cdot mm^{-2}\cdot nm^{-1}$)

[$n_\lambda = t(\lambda)\frac{\lambda T_\lambda}{V(\lambda)}2.24\times 10^3$ quanta per second per square deg per nm ($s^{-1}\cdot \deg^{-2}\cdot nm^{-1}$)]

λ wavelength in nanometers (nm)

$t(\lambda)$ transmittance (0 to 1.0)

$T_\lambda = L_\lambda p$ retinal illumination in trolands

L_λ luminance ($cd\cdot m^{-2}$)

p pupil area (mm^2)

Retinal irradiance (derived from Wyszecki & Stiles 1982)

$\varepsilon_\lambda = t(\lambda)\frac{\lambda T_\lambda}{V(\lambda)}5.261\times 10^{-21}$ watts per square millimeter per nm ($s^{-1}\cdot mm^{-2}\cdot nm^{-1}$)

References

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Stockman, A. and Sharpe, L.T. (2000) Spectral sensitivities of the middle- and long-wavelength sensitive cones derived from measurements in observers of known genotype. *Vision Research*, *40*,1711-1737.

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Wyszecki, G., & Stiles, W. S. (1982). Color Science: concepts and methods, quantitative data and formulae. (2nd ed.). New York: Wiley.